Boundary Conditions and Frequency Equations

The frequency equations have been obtained by employing the boundary conditions for clamped (C) and simply supported (S) plates.

C-S-C-S Plates

Applying the boundary conditions of Eq. (11) for clamped edges $X=0$ and $X=1$, one gets

$$[A_1][C]=[0] \tag{17}$$

where $[A_1]$ is a $4\times(n+5)$ and $[C]$ an $(n+5)\times 1$ matrix. Thus, the frequency equation for C-S-C-S plates is

$$|B/A_1|=0 \tag{18}$$

S-S-S-S Plates

Employing the boundary conditions of Eq. (11) for simply supported edges $X=0$ and $X=1$, one gets the boundary equations for S-S-S-S plate as

$$[A_2][C]=0 \tag{19}$$

where $[A_2]$ is a $4\times(n+5)$ and $[C]$ an $(n+5)\times 1$ matrix.

Hence, the frequency equation for an S-S-S-S plate is

$$|B/A_2|=0 \tag{20}$$

Results and Discussion

Frequency equations (18) and (20) are transcendental in λ^2 from which an infinite number of roots can be determined. The frequencies λ corresponding to the first two modes of vibration of C-S-C-S and S-S-S-S orthotropic rectangular plates have been computed for various values of a/b, α, and β. The orthotropic material parameters are taken as[4]

$$E_2^*/E_1^*=0.32,\quad E^*/E_1^*=0.04,\quad G_0/E_1^*=0.09$$

It is concluded from Figs. 1 and 2 that the frequencies in the first two modes of vibration decrease with the increase in the temperature constant α or taper constant β for both boundary conditions. Further, from Fig. 3, one observes that the frequencies in the first two modes of vibration increase with the increase in aspect ratio a/b for both boundary conditions. It can also be observed from Figs. 1-3 that frequencies in the C-S-C-S plate are higher than in the S-S-S-S plate. λ^2 for $\alpha=\beta=0$ and $a/b=0.5$ and 1.0 is also calculated for the S-S-S-S plate and compared with Dickinson[3] and Hearmon.[4] The values obtained are in close agreement.

References

[1]Leissa, A. W., "Vibration of Plates," NASA SP160, 1969.

[2]Elishakoff, I. B., "Vibration Analysis of Clamped Square Orthotropic Plate," *AIAA Journal*, Vol. 12, 1974, pp. 921-924.

[3]Dickinson, S. M., "The Flexural Vibration of Rectangular Orthotropic Plates," *Applied Mechanics, Transactions of the ASME*, March 1969, pp. 101-106.

[4]Hearmon, R. F. S., "The Frequency of Flexural Vibration of Rectangular Orthotropic Plates with Clamped or Supported Edges," *Journal of Applied Mechanics, Transactions of the ASME*, Dec. 1959, pp. 537-540.

[5]Hoff, N. J., *High Temperature Effect in Aircraft Structures*, Pergamon Press, New York, 1958, p. 62.

[6]Fanconneau, G. and Marangoni, R. D., "Effect of a Thermal Gradient on the Natural Frequencies of a Rectangular Plate," *International Journal of Mechanical Sciences*, Vol. 12, 1970, pp. 113-122.

[7]Tomar, J. S. and Gupta, A. K., "Thermal Effect on Frequencies of an Orthotropic Rectangular Plate of Linearly Varying Thickness," *Journal of Sound and Vibration*, Vol. 90, 1983, pp. 325-331.

[8]Tomar, J. S. and Gupta, A. K., "Thermal Effect on Axisymmetric Vibrations of an Orthotropic Circular Plate of Variable Thickness," *AIAA Journal* (to be published).

A Simple and Accurate Expression for the Viscosity of Nonpolar Diatomic Gases up to 10,000 K

G. Mazor,* G. Ben-Dor,† and O. Igra‡

Ben-Gurion University of the Negev, Beer Sheva, Israel

Introduction and Theoretical Background

FOR any analytical or numerical solution of a viscous flowfield, an expression for the fluid viscosity is required. The viscosity normally enters the analysis through the Reynolds number. In turn, this number characterizes the type of flow (laminar or turbulent). The Reynolds number is an essential parameter in estimating the viscous forces when a solid body is immersed in the fluid. The Reynolds number is needed for evaluating the drag coefficient C_D of the solid body and the Nusselt number N_u. Both of these parameters are necessary for energy and momentum transfer calculations.

Therefore, it is advantageous to express the fluid viscosity as a function of temperature. The simpler this expression can be, the more convenient its use can be in any analytical or numerical solution of a given flowfield.

A variety of theoretically derived expressions for the gas viscosity can be found in Ref. 1. Unfortunately, most of them are too complicated to be readily used in either analytical or numerical calculations. Probably, the most common expression is the one suggested by van Driest.[2] The derivation of this expression is based on Sutherland's model for viscosity[1] and is

$$\frac{\mu}{\mu_0}=\frac{T_0+S}{T+S}\left(\frac{T}{T_0}\right)^{1.5} \tag{1a}$$

where μ and T are the dynamic viscosity and absolute temperature, respectively, S the so-called Sutherland constant, and μ_0 and T_0 the reference values of the dynamic viscosity and absolute temperature.

Equation (1) can also be written in an alternative way as

$$\mu=a_0T^{1.5}/(T+S) \tag{1b}$$

where $a_0=\mu_0(T_0+S)T_0^{-1.5}$. This equation was found to agree with experimental data in the temperature range from 78 K up to about 1200 K. In order to extend this range Gottlieb and Ritzel[3] modified Eq. (1b). They suggested the following relation between μ and T:

$$\mu=\frac{a_0T^{1.5}}{T+S}\left[1+1.53\times10^{-4}\left(\frac{T}{S}-1\right)^2\right] \tag{2}$$

The addition of the term appearing in the square brackets of Eq. (2) increased the upper temperature limit to 2400 K.

Another familiar and frequently used correlation between the viscosity and temperature is

$$\mu/\mu_0=(T/T_0)^{\omega} \tag{3}$$

where ω is a number within the range $0.5\le\omega\le1$. The value $\omega=0.76$ is probably the most frequently used,[4,5]

$$\mu/\mu_0=(T/T_0)^{0.76} \tag{4}$$

Received Dec. 6, 1983; revision received April 16, 1984.

*Graduate Student, Department of Mechanical Engineering.

†Senior Lecturer, Department of Mechanical Engineering.

‡Associate Professor, Department of Mechanical Engineering.

Table 1 Value of ϵ/k for various nonpolar diatomic gases (Ref. 6, App. C)

Substance	ϵ/k	T for neglecting the last 2 terms of Eq. (6), K
Bromine (Br_2)	507.9	700
Chlorine (Cl_2)	316.0	450
Fluorine (F_2)	112.6	150
Hydrogen (H_2)	59.7	80
Iodine (I_2)	474.2	650
Nitrogen (N_2)	71.4	100
Oxygen (O_2)	106.7	150

A more accurate expression for calculating the viscosity is probably the one obtained by Chapman and Enskog (see Ref. 6, p. 395).

$$\mu = 26.69[(MT)^{0.5}/\sigma^2\Omega_v] \quad [10^{-6} \text{ Poise}] \tag{5}$$

where M is the molecular weight, σ the molecule diameter (in Å), and Ω_v the collision integral. Various empirical relations exist for Ω_v. An expression used frequently for many gases is[7]

$$\Omega_v = A(T^*)^{-B} + Ce^{-(DT^*)} + Ee^{-(FT^*)} \tag{6}$$

for nonpolar diatomic gases (See Ref. 6, p. 396) $A=1.16145$, $B=0.14874$, $C=0.52487$, $D=0.77320$, $E=2.16178$, $F=2.43787$, and

$$T^* = kT/\epsilon \tag{7}$$

where k is the Boltzmann constant and ϵ the characteristic energy (i.e., the depth of the potential energy well, see Ref. 6, p. 395).

Equation (6) is valid in the range $0.3 \le T^* \le 100$. In this range the average error between the prediction of Eq. (5) and experimental data for nitrogen is less than 1%.[7] Consequently, Eq. (5) is the most appropriate to serve as a reference for checking the accuracy of the other proposed expressions for the viscosity dependence on temperature.

The Present Model

The aim of the present study is to develop a simple and accurate expression for the dynamic viscosity of nonpolar diatomic gases such as nitrogen and oxygen. Using Eq. (5) one obtains

$$\mu_0 = 26.69[(MT_0)^{0.5}/\sigma^2(\Omega_v)_0] \tag{8}$$

Dividing Eq. (5) by Eq. (8) results in

$$\mu/\mu_0 = (T/T_0)^{0.5}[(\Omega_v)_0/\Omega_v] \tag{9}$$

If the gas temperature is beyond the minimum temperature listed in Table 1 (e.g., for nitrogen $T>100$ K), the last two terms of Eq. (6) are significantly smaller than the first one and hence can be neglected. Thus,

$$(\Omega_v)_0/\Omega_v = (T/T_0)^B \tag{10}$$

Inserting Eq. (10) into Eq. (9) yields

$$\mu/\mu_0 = (T/T_0)^{0.5+B}$$

Using the value $B=0.14874$ finally results in

$$\mu/\mu_0 = (T/T_0)^{0.64874} \tag{11}$$

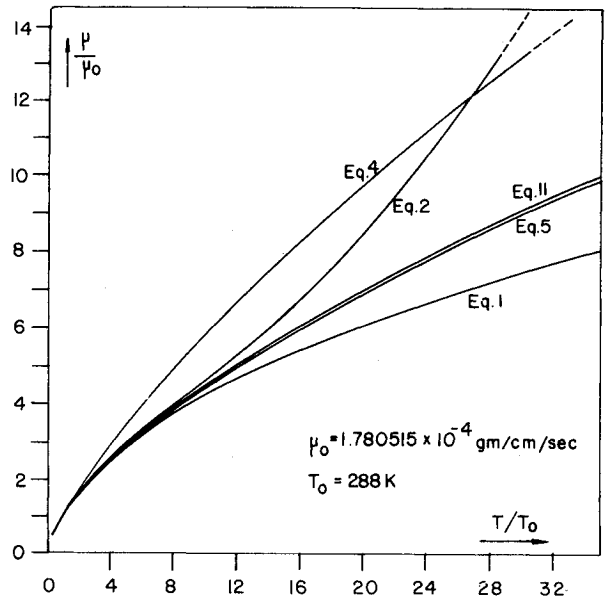

Fig. 1 Comparison of empirical and experimental results for the dynamic viscosity of nitrogen (78-10,200 K).

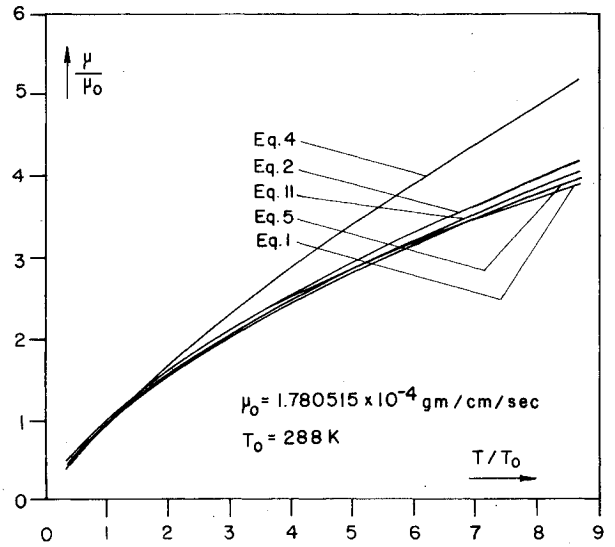

Fig. 2 Comparison of empirical and experimental results for the dynamic viscosity of nitrogen (78-2,700 K).

Results and Discussion

Equations (1), (2), (4), (5), and (11) were all plotted in the $(\mu/\mu_0, T/T_0)$ plane. The results are shown in Figs. 1 and 2 (μ_0 and T_0 are for nitrogen).

As mentioned earlier, the viscosity as obtained by Chapman and Enskog's model[6] along with the Neufeld-Janzen-Aziz[7] expression for Ω_v [Eqs. (6) and (7), respectively] were found to deviate from experimentally obtained values by less than 1% for $T^* < 100$. Consequently, the curves labeled as Eq. (5) in Figs. 1 and 2 are used as a reference for the comparison with the other models.

Sutherland's model [Eq. (1a)] agrees with the reference equation (5) up to about $T=1800$ K. Increasing the temperature T beyond 1800 K results in smaller values for the dynamic viscosity μ, as compared with experimental results. The disagreement becomes progressively worse with increasing temperatures.

Gottlieb and Ritzel's model [Eq. (2)] gives values almost identical to those of Sutherland's model [Eq. (1a)] up to $T=1800$ K. However, in the range $1800 < T < 2400$ K, it shows

better agreement with the reference line [Eq. (5)]. As the temperature increases beyond 2400 K, and in accordance with their claim,[3] the predictions of their model shift away from the experimental data very rapidly to result in much higher values for the gas viscosity.

The results obtained using Eq. (4) agree writh the experimental data only up to about $T=600$ K. Above this temperature, Eq. (4) predicts higher values than the experimentally obtained values represented by the curves labeled as Eq. (5).

The presently proposed simple expression for μ, given by Eq. (11), agrees very well with the reference line [Eq. (5)] up to 10,000 K.

Conclusions

An expression relating the dynamic viscosity of nonpolar diatomic gases to the absolute temperature has been proposed. This expression is very simple, easy to use, and gives accurate values for μ at temperatures of 100-10,000 K. Owing to the fact that reference values for μ_0 and T_0 are needed in order to obtain absolute values for μ at given temperatures, the proposed expression is semiempirical. The proposed expression is valid for nonpolar diatomic gases provided that $0.3 \leq T^* < 100$ and that the gas under consideration is not too compressed.

Acknowledgments

The authors would like to thank Prof. J. J. Gottlieb of the Institute of Aerospace Studies, University of Toronto, and Mr. A. Kaniel of the Department of Mechanical Engineering, Ben-Gurion University of the Negev, for their suggestions and assistance during the course of this work.

References

[1] Chapman, S. and Coulling, T. G., *The Mathematical Theory of Non-Uniform Gases,* 3rd ed., Cambridge University Press, Cambridge, England, 1970.

[2] van Driest, E. R., "Investigation of Boundary Layer in Compressible Fluids Using the Crocco Method," NACA TN 2577, 1952.

[3] Gottlieb, J. J. and Ritzel, D. V., "A Semi-Empirical Equation for the Viscosity of Air," DRES, Ralston, Canada, Tech. Note 454, 1979.

[4] Schlichting, H., *Boundary Layer Theory,* 4th ed., McGraw-Hill Book Co., New York, 1962.

[5] Glass, I. I. and Hall, J. G., *Handbook of Supersonic Aerodynamics,* Sec. 18, "Shock Tubes," NAVORD Rept. 1488, Vol. 6, Dec. 1959.

[6] Reid, R. C., Prausnitz, J. M., and Sherwood, T. K., *The Properties of Gases and Liquids,* 3rd ed., McGraw-Hill Book Co., New York, 1977.

[7] Neufeld, P. D., Janzen, A. R., and Aziz, R. A., "Empirical Equations to Calculate 16 of the Transport Collision Integrals $\Omega^{(l,s)^*}$ for the Lennard-Jones (12-6) Potential," *The Journal of Chemical Physics,* Vol. 57, No. 3, Aug. 1972, pp. 1100-1102.